## DAMAGE IN LIQUEFACTION - A PROBABILISTIC MODEL

by

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# ABSTRACT

The damage associated with earthquake-induced liquefaction is of major concern to engineers. To estimate damage it is necessary to go beyond the evaluation of liquefaction potential. A minimum volume of sand must undergo considerable strain due to liquefaction to produce noticeable damage at the referenced location. The properties of the soil in this volume also need to be modeled appropriately. A threedimensional probabilistic model to evaluate liquefaction potential has already been developed considering the effects of compliance, sample preparation methods, mean grain size, multidirectional shaking and some other secondary factors. Soil liquefaction following earthquakes causes settlement, tilting, sliding, etc. at the site. The damage criterion related to differential settlement or rotation of foundation is specifically addressed in this paper. For a given liquefied volume, the risk of liquefaction as well as maximum rotation are estimated. Subsidence theory is used to estimate the rotation. Using very conservative assumptions, the risk of different amounts of rotation is estimated. The risk of exceeding a given amount of rotation depends on the Standard Penetration Test values for the deposit as well as the subsidence factor. This is a first step in assessing damage in liquefaction.

### INTRODUCTION

There are hundreds of recent cases of ground failure and damage to structures due to liquefaction during earthquakes in China, Japan, Yugoslavia, Chile, Central America and the United States. During the 1964 earthquake in Niigata, Japan, many structures settled several feet and suffered up to 80 degrees of tilting (1,2). The same year, in Valdez, Alaska, extensive flow slides washed entire sections of the waterfront into the sea. Numerous studies have been conducted since then to understand the behavior of cohesionless soil under earthquake loading. Researchers are investigating the causes of the problem; however, the damage associated with liquefaction is a major problem facing an engineer.

To estimate damage during liquefaction, it is necessary to go one step beyond the evaluation of liquefaction potential. Since a volume of sand has to undergo a considerable amount of strain to produce a noticeable amount of damage at the referenced location, it is very important to identify this critical volume. The properties of the soil in this volume also need to be modeled appropriately. It is known that liquefaction does not always lead to damage and that initial deposit conditions affect the extent of damage. Limiting or eliminating damage during liquefaction would be a reasonable criterion for this type of approach. So far, the direct evaluation of damage has not received proper attention. The purpose of this paper is to evaluate damage associated with liquefaction considering differential settlement or angle of rotation as the design criterion.

# PROBLEM DESCRIPTION

The basic cause of liquefaction in a saturated cohesionless soil deposit during an earthquake is the buildup of pore water pressure due to the application of cyclic shear stresses induced by the ground motion. The cohesionless soil tends to become more compact while the soil structure rebounds to the extent necessary to keep the volume constant. This interplay of volume reduction and soil structure rebound determines the magnitude of the increase in pore water pressure. If sufficient pore water pressure is produced, the effective stress becomes zero and the deposit assumes the characteristics of a viscous liquid. This essentially leads to liquefaction.

The liquefaction of a deposit is a very complex problem. Quite a few factors influence the liquefaction potential evaluation. Moreover, each factor influences the evaluation to a different degree. For proper evaluation, information on soil properties affecting the liquefaction phenomenon and earthquake loading needs to be available. The estimation of in situ soil parameters can be obtained by measuring them directly in the deposit, or indirectly from empirical relationships or by measuring them in the laboratory using so-called "undisturbed" samples. Considerable error can be incurred during these processes (3,4,5). The nonhomogeneity of the soil properties in the liquifiable volume has to be modeled in three dimensions. Long-distance fluctuations and local variations in soil properties can only be modeled effectively using probability theory. Seismic loading is also unpredictable (6). This necessitates the availability of a simple but efficient and practical probabilistic model to study the risk of damage associated with the liquefaction phenomenon.

Haldar and Miller (5) developed a three-dimensional probabilistic model to evaluate liquefaction potential considering the effects of compliance, sample preparation methods, mean grain size, multidirectional shaking and some other secondary factors. Three-dimensional soil properties are evaluated using the information on the corresponding scales of fluctuation. For a given seismicity and soil deposit conditions, the model will give the risk of liquefaction for a given volume of sand. Different risks would be obtained for different volumes of sand.

Damage in liquefaction is the subject of this paper. The most common types of damage that can be expected due to liquefaction following an earthquake are settlement, differential settlement, subsidence, and tilting of a structure at the site. Allowable values for these parameters have been discussed extensively in codes, design guidelines, and the literature (7). It is quite logical to assume that the same standards should also be used for assessing liquefaction damage. The damage criterion related to differential settlement or rotation of foundation is specifically addressed in this paper.

The amount of differential settlement or rotation of foundation due to liquefaction depends on the liquefied volume, deposit conditions, depth of the volume from the ground surface, etc. Thus, the amount of differential settlement or rotation can be related to the soil volume. For the same soil volume, the risk of liquefaction can also be estimated as discussed before. Thus, for the same volume, it is possible to estimate the risk associated with different amounts of differential settlement or rotation due to earthquake-induced liquefaction.

It is not possible to consider all possible damage scenarios; however, some limit (extreme) cases can be studied to model soil liquefaction like that which occurred in Alaska and Niigata. One extremely conservative limit case could be treating liquefaction as a subsidence problem. The mathematical model could be developed by modeling liquefaction as a subsidence problem where all the liquefied soil volume has flowed away from beneath the foundation, creating a void. This is a very conservative and simplified approach. It could be developed further to closely resemble the real situation. The settlement due to subsidence will be discussed in the following section.

## SURFACE SUBSIDENCE AND FOUNDATION ROTATION

The problem of predicting ground surface movements due to a void or cavity underneath the surface is not new. Subsidence theory is well developed in mining and tunnel engineering (8,9). Several theories of subsidence have been developed. They can be broadly classified into two types, empirical and phenomenological.

The empirical approaches are based on observations of ground movement without particular regard to the mechanics of subsidence (9). Empirical models relating vertical surface subsidence to the type of subsurface material, dimensions of the void and depth of the void below the surface have proved to yield reliable results for mining engineering purposes (8,9). The advantages of the empirical approaches are simplicity and reliability, but development of these models requires a considerable amount of field observations, and extrapolation from one set of subsurface conditions to another is difficult. Phenomenological models are based on the principles of continuum mechanics. Included in this classification are the classical elasticity models, viscoelastic, and plastic models (9). The advantages of these approaches are that they attempt to model the underlying mechanics of subsidence, and extrapolation to other subsurface conditions is easier than with the empirical approaches. However, they are extremely complicated, the soil parameters that are needed for input are very difficult if not impossible to measure with precision and there is some doubt as to whether sufficiently reliable results can be obtained by representing a granular material like a cohesionless soil by a continuum. Since the phenomenological approaches are still in the development stage and the empirical approaches have gained wide acceptance in the profession (8), an empirical approach called the method of influence functions is used in this study.

The method of influence functions is based on the notion that an infinitesimal void element beneath the surface produces a small subsidence at the surface; the total surface subsidence is then given by the sum of the small subsidences of an infinite number of infinitesimal void elements encompassing the void (8). Several influence functions have been proposed for different subsurface conditions (8). The most appropriate influence function for a cohesionless soil deposit can be represented by Eq. 1 (9) and is shown in Fig. 1.

$$f(\mathbf{x'},\mathbf{x}) = \frac{S_{\max}}{B} \cdot \exp\left[-\pi \left(\frac{\mathbf{x'}-\mathbf{x}}{B}\right)^2\right]; \quad |\mathbf{x'}-\mathbf{x}| \leq B$$
(1)

#### = 0 ; elsewhere

in which  $S_{max}$  = the largest amount of settlement for a void of thickness m regardless of the plan dimensions of the void; and B = the critical length, as shown in Fig. 1. B is the value of the width of a soil volume, L, for which only one point at the ground surface settles the maximum amount,  $S_{max}$ . For each depth h, there is a unique value for B. Using Eq. 1, the total settlement, S(x), at a point x on the surface is given by

$$S(x) = \int_{-L}^{L} f(x', x) dx'$$
 (2)

The maximum settlement S  $_{\rm max}$  in Eq. 1 for horizontal rectangular voids can be represented as (8)

 $S_{max} = am$  (3)

in which m = void thickness; and a = the subsidence factor. The subsidence factor, although considered a constant for a subsurface material in many applications, depends on the unit weights of the soil before and after subsidence and the depth-to-thickness ratio,

h/m. Field observations have also indicated that the point of zero subsidence, point C in Fig. 1, is located a horizontal distance B from a vertical plane passing through the edge of the void (8).

Differential settlement or rotation may be a better indicator of the damage than the total settlement in many cases. Using Eq. 2, the slope of the foundation settlement,  $\rho(x)$ , can be shown to be:

$$\rho(\mathbf{x}) = \left| \frac{\mathrm{d}}{\mathrm{d}\mathbf{x}} \int_{-\mathrm{L}}^{\mathrm{L}} \mathbf{f}(\mathbf{x}',\mathbf{x}) \, \mathrm{d}\mathbf{x}' \right|$$
(4)

Substituting Eq. 1 in Eq. 4, the maximum value of the slope can be shown to be

$$\rho_{\max} = \frac{S_{\max}}{B} = \frac{a_{\max}}{B}$$
(5)

The critical length parameter B is given by

 $B = h \cot \alpha \tag{6}$ 

in which  $\alpha$  = the limit angle of influence, as shown in Fig. 1. Terzaghi (10) considered a problem similar to this one in connection with arching action in soils. The angle  $\alpha$  may approach 90° for very small values of h; however, it approaches a value of  $45^{\circ} + \phi/2$  for large values of h.  $\phi$  is the angle of internal friction for the soil. Typical  $\alpha$  and  $\phi$  values are given in Table 1.

The subsidence factor a can be estimated as

$$a = \begin{bmatrix} \frac{3}{2} & \frac{\gamma_f - \gamma_o}{\gamma_f} & \cdot & \frac{h}{m} \end{bmatrix} + 1$$
(7)

in which  $\gamma_{_{\mbox{\scriptsize 0}}}$  and  $\gamma_{_{\mbox{\scriptsize f}}}$  = the unit weight of soil before and after the

subsidence has occurred. Eq. 7 yields similar values to those given by Brauner (8). The values of a are expected to be between 0.1 and 0.9 for sand deposits.

#### RESULTS

The ground surface rotations produced by a subsurface void are given by Eqs. 5, 6, and 7. Thus, with Eqs. 5, 6, and 7 the risk of rotation can be found from the risk of liquefaction of a soil volume. Haldar and Miller (5) described a site (SITE A) where the risks of liquefaction are estimated for different soil volumes and Standard Penetration Test (SPT) values. The same site is considered here. The details of the site can not be given here due to lack of space. A soil volume of 100' x 100' x m is considered here.  $\phi$  is assumed to be 36°. For a = 0.1, 0.2, and 0.6, the amount of rotation as well as the risk of liquefaction is calculated for SPT values of 6 and 15. The risk versus rotation is plotted in Fig. 2. It must be pointed out here that the  $\rho_{\mbox{max}}$  values in Fig. 2 are upper

bound estimates of rotation. The total liquefied volume is not expected to flow away beneath the foundation creating a void. Thus, the a values that need to be considered to estimate  $\rho_{\max}$  would be much

smaller than in the pure subsidence problem. In addition,  $\rho_{\text{max}}$  is the

maximum slope of the subsidence curve occurring at a point. In reality, angular distortion, the differential settlement divided by the foundation length, is a better measure of damage. Thus,  $\rho_{max}$ 

always overpredicts the level of damage.

Some interesting observations can be made from Fig. 2. The risk of rotation depends on the SPT values and the subsidence factor. As the site becomes denser, the risk of a given amount of rotation decreases. Also, the subsidence factor contributes significantly to the estimation of rotation. A lower value of a is expected due to earthquake-induced liquefaction. The most appropriate value of a needs to be calibrated using case studies. This work is now in progress. It can also be observed from Fig. 2 that the risk decreases as the value of the tolerable rotation increases. The tolerable rotation for ordinary buildings, generally accepted by the profession is 1/300 (7). For the problem under consideration, the risks of 1/300 rotation for SPT values of 6 and 15 are  $1.3 \times 10^{-2}$  and  $3.5 \times 10^{-3}$ , respectively. It is also interesting to note that as the value of acceptable rotation.

#### ACKNOWLEDGMENT

This material is based upon work partly supported by the National Science Foundation under Grant No. CE-811691. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the writers and do not necessarily reflect the views of the National Science Foundation.

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Table 1. -  $\phi$  and  $\alpha$  Values for Sand Deposits (from Ref. 11)

So11	Condition	Relative Density	φ	α
Very	Loose Sand	< 20%	< 29°	< 59.5°
	Loose Sand	20 - 40%	29 – 30°	59.5 - 60°
	Medium Sand	40 - 60%	30 – 36°	60 - 63°
	Dense Sand	60 - 80%	36 - 41°	63 - 65.5°
Very	Dense Sand	> 80%	> 41°	> 65.5°





